Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number

Candidate Number

Pearson Edexcel Level 3 GCE

Paper reference **8FM0/26**

Further Mathematics

Advanced Subsidiary
Further Mathematics options
26: Further Mechanics 2
(Part of option J)

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear.
- Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a value of g is required, take $g = 9.8\,\text{m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 4 questions.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

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1.

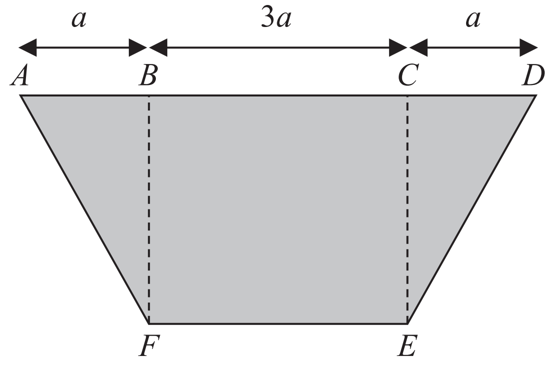


Figure 1

A uniform plane lamina is in the shape of an isosceles trapezium *ABCDEF*, as shown shaded in Figure 1.

- *BCEF* is a square
- $AB = CD = a$
- $BC = 3a$

(a) Show that the distance of the centre of mass of the lamina from *AD* is $\dfrac{11a}{8}$

(5)

The mass of the lamina is M

The lamina is suspended by two light vertical strings, one attached to the lamina at *A* and the other attached to the lamina at *F*

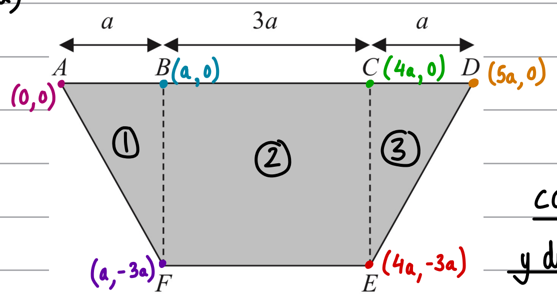
The lamina hangs freely in equilibrium, with *BF* horizontal.

(b) Find, in terms of M and g, the tension in the string attached at *A*

(2)

Question 1 continued

a)



Taking Point A with coordinates (0,0) as the origin

COM of triangle 1 from A

y direction: $\frac{0+0+-3a}{3} = -a$

x direction: $\frac{0+a+a}{3} = \frac{2a}{3}$

COM of square 2 from A

Since shape is symmetrical in multiple axes it will be at the centre $\frac{3a}{2}$ from B in positive x direction and negative y direction.

COM of 2 at: $\left(\frac{5a}{2}, -\frac{3a}{2}\right)$

COM of triangle 3 from A

y direction: $\frac{0+0+-3a}{3} = -a$

x direction: $\frac{4a+5a+4a}{3} = \frac{13a}{3}$

Overall COM of lamina from AD

Since it is from AD, only consider y-axis. Area is proportional to the mass as the lamina is uniform, we can substitute the force due to the mass with the area.

moments = force x perpendicular distance
The sum of moments is equal to the overall moment acting through the COM.
Mathematically → $\sum m_i x_i = \bar{x} \sum m_i$
where m = force
x = perpendicular distance

(Area of triangle 1 x COM) + (Area of square 2 x COM) + (Area of triangle 3 x COM) = Area of lamina x COM

$$\tfrac{1}{2}(3a)(a) \times (-a) + (3a)(3a) \times \left(-\tfrac{3a}{2}\right) + \tfrac{1}{2}(a)(3a) \times (-a) = \left(\tfrac{1}{2}(3a)(a) + (3a)(3a) + \tfrac{1}{2}(a)(3a)\right) \times \bar{x}$$

$$-\frac{3a^3}{2} - \frac{27a^3}{2} - \frac{3a^3}{2} = 12a^2\,\bar{x}$$

$$-\frac{33}{2}a^3 = 12a^2\,\bar{x}$$

$$\bar{x} = -\frac{11}{8}a$$

$\therefore$ Distance of COM from $AD = \frac{11}{8}a$

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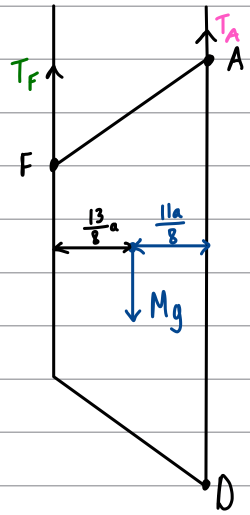
Turn over

Question 1 continued

b)



Taking moments about F

T_A = tension on string at A

$\frac{13}{8}a \times Mg = T_A \times 3a$

$\boxed{T_A = \frac{13}{24} Mg}$

Using formulae

sum of clockwise moments = sum of anti-clockwise moments

where moments = force × perpendicular distance

Question 1 continued

(Total for Question 1 is 7 marks)

2.

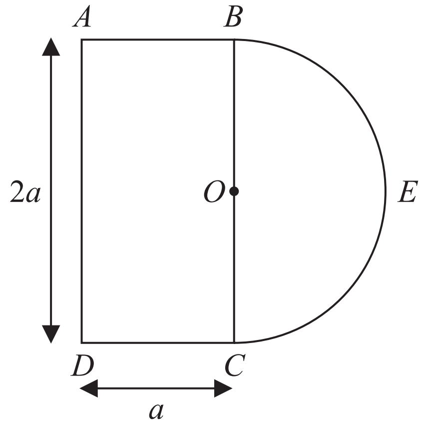


Figure 2

Uniform wire is used to form the framework shown in Figure 2.

In the framework

- $ABCD$ is a rectangle with $AD = 2a$ and $DC = a$
- BEC is a semicircular arc of radius a and centre O, where O lies on BC

The diameter of the semicircle is BC and the point E is such that OE is perpendicular to BC.

The points A, B, C, D and E all lie in the same plane.

(a) Show that the distance of the centre of mass of the framework from BC is

$$\frac{a}{6+\pi}$$

(5)

The framework is freely suspended from A and hangs in equilibrium with AE at an angle $\theta°$ to the downward vertical.

(b) Find the value of θ.

(4)

The mass of the framework is M.

A particle of mass kM is attached to the framework at B.

The centre of mass of the loaded framework lies on OA.

(c) Find the value of k.

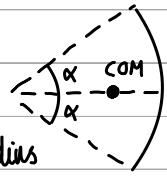
(3)

Question 2 continued

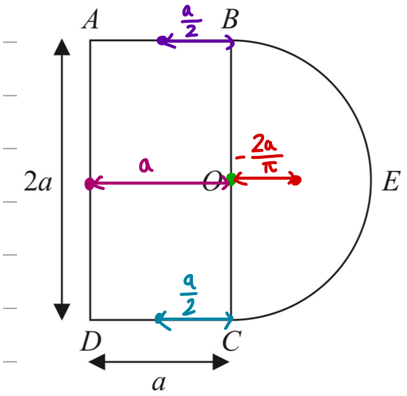
a)

Formulae for distance of centre of mass of a uniform circular arc from its centre

$\dfrac{r\sin\alpha}{\alpha}$ from centre



where α is in radians and r = radius

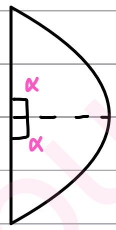


Applying formulae

$\alpha = 90° = \dfrac{\pi}{2}$ rad

$\therefore$ distance from O

$= \dfrac{a\sin\frac{\pi}{2}}{\frac{\pi}{2}} = \dfrac{2a}{\pi}$



To find COM from BC, take moments about BC in x direction.

Taking direction $\overrightarrow{BA}$ as positive

$a \times \frac{a}{2} + a \times \frac{a}{2} + 2a \times a + 2a \times 0 + a\pi \times -\frac{2a}{\pi} =$

$(a + a + 2a + 2a + a\pi)\,\bar{x}$

$\frac{a^2}{2} + \frac{a^2}{2} + 2a^2 + 0 + -2a^2 = (6a + a\pi)\bar{x}$

$a^2 = a(6+\pi)\bar{x}$

$\boxed{\bar{x} = \dfrac{a}{6+\pi}}$ => so distance from BC is proven

Since this is a framework, we use

moment = force × perpendicular distance

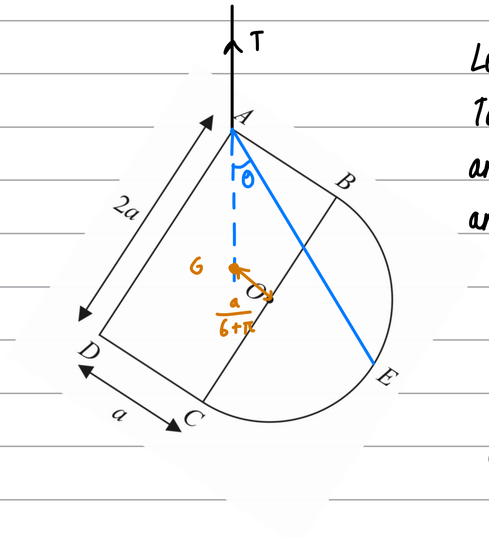
force is proportional to the lengths as they are uniform wire so we can replace force with length since we use

$\sum m_i x_i = \bar{x} \sum m_i$

where sum of moments of each component is equal to the singular moment through the COM

Question 2 continued

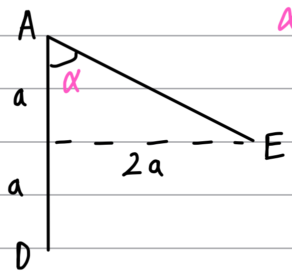
b)



Let G = COM

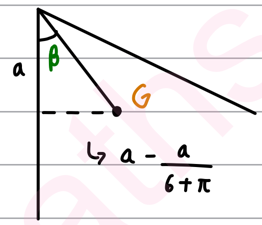
To find θ, we must find angle DAE and subtract angle DAG from it

Angle DAE



$$\alpha = \tan^{-1}\left(\frac{2a}{a}\right) = \tan^{-1}(2)$$

Angle DAG



$$\beta = \tan^{-1}\left(\frac{a - \frac{a}{6+\pi}}{a}\right)$$

$$\beta = \tan^{-1}\left(1 - \frac{1}{6+\pi}\right)$$

$$\therefore \theta = \alpha - \beta$$

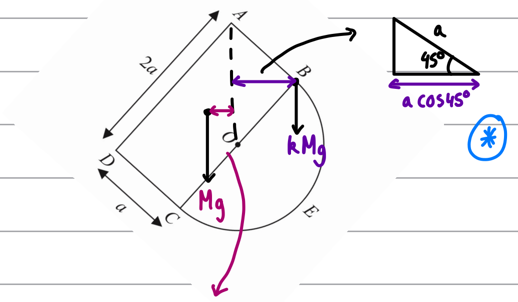
$$= \tan^{-1}(2) - \tan^{-1}\left(1 - \frac{1}{6+\pi}\right)$$

$$\theta = 21.7^\circ \ (3sf)$$

Question 2 continued

c)

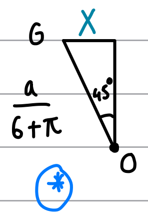
Taking moments about OA



$$Mg \times \frac{a}{6+\pi} \sin 45 = kMg \times a\cos 45$$

$$\frac{Mg\,a}{6+\pi} = kMg\,a$$

$$\therefore\ k = \frac{1}{6+\pi}$$



$$X = \frac{a}{6+\pi}\sin 45$$

(*) Using trigonometry to obtain perpendicular distances

(Total for Question 2 is 12 marks)

3. A cyclist is travelling around a circular track which is banked at an angle α to the horizontal, where $\tan\alpha = \frac{3}{4}$

The cyclist moves with constant speed in a horizontal circle of radius r.

In an initial model,

- the cyclist and her cycle are modelled as a particle
- the track is modelled as being rough so that there is sideways friction between the tyres of the cycle and the track, with coefficient of friction μ, where $\mu < \frac{4}{3}$

Using this model, the maximum speed that the cyclist can travel around the track in a horizontal circle of radius r, without slipping sideways, is V.

(a) Show that $V = \sqrt{\dfrac{(3+4\mu)rg}{4-3\mu}}$

(7)

In a new simplified model,

- the cyclist and her cycle are modelled as a particle
- the motion is now modelled so that there is **no** sideways friction between the tyres of the cycle and the track

Using this new model, the speed that the cyclist can travel around the track in a horizontal circle of radius r, without slipping sideways, is U.

(b) Find U in terms of r and g.

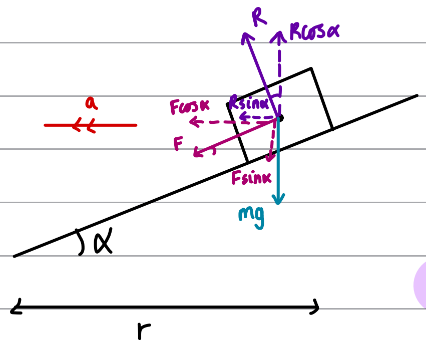
(2)

(c) Show that $U < V$.

(2)

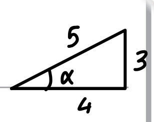
Question 3 continued

a)



where F = friction
$= \mu R$

$\tan\alpha = \frac{3}{4}$



$\sin\alpha = \frac{3}{5}$

$\cos\alpha = \frac{4}{5}$

Equation for centripetal acceleration : $a = \frac{v^2}{r}$

Resolving forces horizontally

$R(\rightarrow)$: $R\sin\alpha + F\cos\alpha = \frac{mV^2}{r}$

$\frac{3}{5}R + \frac{4}{5}\mu R = \frac{mV^2}{r}$

$R\left(\frac{3}{5} + \frac{4}{5}\mu\right) = \frac{mV^2}{r}$

Substituting 1

$\frac{5mg}{4-3\mu}\left(\frac{3}{5} + \frac{4}{5}\mu\right) = \frac{mV^2}{r}$

$\frac{3\cancel{m}g + 4\mu\cancel{m}g}{4-3\mu} = \frac{\cancel{m}V^2}{r}$

$\frac{(3+4\mu)rg}{4-3\mu} = V^2$

$V = \sqrt{\frac{(3+4\mu)rg}{4-3\mu}}$

Resolving forces vertically

$R(\downarrow)$: $R\cos\alpha - F\sin\alpha = mg$

$\frac{4}{5}R - \frac{3}{5}\mu R = mg$

$R\left(\frac{4}{5} - \frac{3}{5}\mu\right) = mg$

$R = \frac{5mg}{4-3\mu}$ 1

b) No sideways friction so $\mu = 0$

when $\mu = 0$

$U = \sqrt{\frac{3rg}{4}}$

c) Comparing numerators and denominators of each:

numerators: $3 + 4\mu > 3$
(u) (v)

denominators: $4 - 3\mu < 4$
(u) (v)

so $\frac{3}{4} < \frac{3+4\mu}{4-3\mu}$

hence $U < V$

Turn over

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Question 3 continued

Question 3 continued

(Total for Question 3 is 11 marks)

4. A particle P moves on the x-axis. At time t seconds the velocity of P is $v\,\text{m s}^{-1}$ in the direction of x increasing, where

$$v = \frac{1}{2}\left(3e^{2t} - 1\right) \qquad t \geqslant 0$$

The acceleration of P at time t seconds is $a\,\text{m s}^{-2}$

(a) Show that $a = 2v + 1$

(2)

(b) Find the acceleration of P when $t = 0$

(1)

(c) Find the exact distance travelled by P in accelerating from a speed of $1\,\text{m s}^{-1}$ to a speed of $4\,\text{m s}^{-1}$

(7)

a) $a = \dfrac{dv}{dt}$ $\quad\therefore\quad a = \dfrac{d\left(\frac{1}{2}\left(3e^{2t}-1\right)\right)}{dt}$

$= \frac{1}{2}\left(6e^{2t}\right) = 3e^{2t}$

since $v = \frac{1}{2}\left(3e^{2t} - 1\right)$

$2v = 3e^{2t} - 1$

$3e^{2t} = 2v + 1$

Rearranging for $a = 3e^{2t}$, we get $a = 2v + 1$

b) at $t = 0$, $a = 3e^{0} = 3\text{ms}^{-2}$

Substituting into expression

Question 4 continued

c)

$V = \frac{dx}{dt}$ Substituting $\frac{1}{2}\left(3e^{2t}-1\right) = \frac{dx}{dt}$

$$\int \frac{1}{2}\left(3e^{2t}-1\right)dt = \int 1\,dx$$

$$\frac{1}{2}\left(\frac{3}{2}e^{2t}-t\right) + C = x$$

where C is a constant

$$\frac{3}{4}e^{2t} - \frac{t}{2} + C = x$$

Substitute values to obtain ↓ values for time 't'

x between two times $= \left[\frac{3}{4}e^{2t} - \frac{t}{2}\right]_a^b$

No need to find constant C

$V=1$: $1 = \frac{1}{2}\left(3e^{2t}-1\right)$ $\quad$ $V=4$: $4 = \frac{1}{2}\left(3e^{2t}-1\right)$

$3 = 3e^{2t}$ $\quad$ $9 = 3e^{2t}$

$1 = e^{2t}$ $\quad$ $3 = e^{2t}$

$2t = 0$ $\quad$ $2t = \ln 3$

$t = 0$ $\quad$ $t = \frac{1}{2}\ln 3$

Using yellow highlighted expression

$$x = \left[\frac{3}{4}e^{2t} - \frac{t}{2}\right]_0^{\frac{1}{2}\ln 3}$$

$$= \left(\frac{3}{4}e^{2\left(\frac{1}{2}\ln 3\right)} - \frac{\frac{1}{2}\ln 3}{2}\right) - \frac{3}{4}$$

$$= \frac{3}{4}(3) - \frac{1}{4}\ln 3 - \frac{3}{4}$$

$$= \frac{3}{4} - \frac{1}{4}\ln 3 \quad m$$

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Question 4 continued

(Total for Question 4 is 10 marks)

TOTAL FOR FURTHER MECHANICS 2 IS 40 MARKS